Mathematics: The Critical Filter

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Mathematics education has received wide attention from educators and policymakers in recent years because of the increasingly important role it plays in our technological world. Competence in mathematical skills is an important qualification for admission to many college and university programs, and in turn to high paying occupations. Thus, it is often called the “critical filter”. The provincial Ministries of Education monitor the proficiency of students in mathematics through participation in a number of national and international assessments. Research at the Atlantic Centre for Policy Research is examining differences among provinces in their achievement in an attempt to understand inequities between males and females, and among students with differing socioeconomic backgrounds. The research also aims to understand why some schools and schooling systems tend to perform better than others.

< Canadian students fared better than U.S. students in the 1994 Third International Study of Mathematics and Science (TIMSS); however, their counterparts in many European countries scored considerably better.

< Within Canada, students in Quebec have consistently performed better in national and international studies than students in other provinces. Provincial differences in achievement are evident as early as grade 2, and increase throughout the elementary years. By the end of grade 7, there are large disparities among the provinces across all domains of mathematic achievement.

< Students from low socioeconomic backgrounds have lower scores on average than children from high socioeconomic backgrounds. Questions in the domains of proportionality, fractions, and statistics seem to be particularly difficult for children from less advantaged backgrounds.

< Females and males had similar scores in four of the six domains of mathematics tested at grades 7 and 8 in the TIMSS. Males scored somewhat better on items associated with measurement, and considerably better on items pertaining to proportionality.

< A disproportionate number of females drop out of advanced mathematics in their final year of high school, even though their achievement scores and participation rates are equal to or better than that of males through to the end of grade 11. University and secondary school staff may need to inform high school students about the ramifications of withdrawing from mathematics too early, thereby limiting their future opportunities.
The Critical Filter

For several decades mathematics has been called the “critical filter”, because students who are inadequately prepared in mathematics during secondary school lose many of the career choices that would otherwise be available to them. Also, for those students who pursue post-secondary education, success in first-year calculus is a prerequisite for most degree programs in business, computing, and the sciences. Moreover, the role of mathematics in filtering students for high-paying jobs and long-term employment is likely to increase. During the 1970’s and 1980’s, employment in low-technology, low-wage industries decreased across all countries in the Organisation for Economic Co-operation and Development (OECD), and employment projections for the next decade indicate an increased demand for moderately skilled technical and administrative workers and highly skilled professionals (OECD, 1995). The new high-technology jobs will require an understanding of computerized data analyses and sophisticated mathematical models.

Despite the increased demand for mathematical skills in the workplace, youth in Canada and the U.S. have not fared well in international comparisons of mathematics achievement. In the TIMSS, grade eight students in 10 of the participating countries outperformed Canadian students, and students in the U.S. scored even lower. Canada and the U.S. have been able to compete successfully in the global economy, probably because they have a disproportionate number of graduates with skills at the top end of the distribution. However, their future economic success may be compromised if they are unable to achieve a high level of quantitative proficiency across a wide base of their populations. Research being conducted at the Atlantic Centre for Policy Research, in co-operation with Human Resources Development Canada and Statistics Canada, is examining data from several national and international studies. The aims of the research are to describe the distribution of quantitative skills among various subgroups, and to discern which policies and practices might lead to greater proficiency in mathematics skills.

The Quantitative Literacy Skills of Canadian Youth

In 1994, Canada was among the first eight countries that participated in the International Adult Literacy Study (IALS). Since then, the IALS has been extended to 24 countries. The study entails in-depth interviewing and testing of a representative sample of adults aged 16 and older in each country. The IALS tests reflect a broad definition of literacy, and include items pertaining to prose, document, and quantitative literacy. The quantitative test requires respondents to locate, comprehend, and utilize mathematical ideas embedded in text, and thus is a more general measure of quantitative skills than the curriculum-specific tests used in the TIMSS.

Figure 1 displays the results for 12 countries that had participated by 1996. Canadian youth aged 16 to 25 scored lower than Sweden, Switzerland, Netherlands, Germany, and Belgium. The disparities are immense: a Canadian youth whose parents had completed a grade 12 education, on average, scored about 20% of a standard deviation above the international mean; whereas in the high-scoring European countries, a youth whose parents had a similar education scored about 40 to 60% of a standard deviation above the international mean. A difference of 15% on these tests is approximately equivalent to one additional year of schooling. Thus the gap in quantitative skills is roughly equivalent to about two additional years of schooling (Willms, 1997).

The relationship between a social outcome, such a literacy skills, and factors describing people’s socioeconomic background, such as the level of parental education, is usually referred to as a socioeconomic gradient. An important question concerning socioeconomic gradients is whether they tend to converge or diverge with increasing levels of socioeconomic status. The socioeconomic gradients displayed in Figure 1 clearly converge, which indicates that the success of a country in achieving high literacy skills overall depends largely on its ability to achieve high literacy skills for its least advantaged youth. An analysis of the socioeconomic gradients for the literacy skills of youth across the ten Canadian provinces revealed a similar pattern: Quebec and the three prairie provinces had relatively shallow gradients, while Ontario, British Columbia, and the four Atlantic provinces had relatively steep gradients (Willms, 1997). Youth from advantaged backgrounds tended to perform well in all provinces, while youth from less advantaged backgrounds performed reasonably well in Quebec and the prairie provinces, but not in the other provinces.
Provincial Differences Appear Early

Some of the variation among provinces in their test performance is attributable to the socioeconomic background of the students. However, results from Canada’s National Longitudinal Study of Children and Youth (NLSCY) reveal large and statistically significant differences among the ten provinces in their mathematics achievement, even after taking account of students’ family background (Willms, 1996). Differences among the provinces appear at the end of grade 2: students in Ontario lagged behind the national average by about one month of schooling; whereas students in New Brunswick, Manitoba, Nova Scotia, British Columbia and Quebec scored about one to four months of schooling above the national average.

By the end of grade 4, the background-adjusted scores for Quebec were about six months of schooling higher than the national average, and by the end of grade 6, it was nearly one full year of schooling higher. These results indicate that differences among the provinces widen as children progress through the elementary grades.

Provincial Differences Across Domains of Mathematics Achievement

Some educators have argued that the provincial differences observed in the IALS and the NLSCY are evident because the tests tap only certain domains of achievement, and these are not emphasized in every province’s curricula. The TIMSS, which was also conducted in 1994, covered mathematics skills in six different domains: fractions, geometry, algebra, statistics, measurement, and proportionality.

In addition to Canada’s participation in TIMSS, five provinces participated as “member countries” in the TIMSS. Our analyses examined the test results for grade 7 and 8 students for these provinces, and for “Other (French)” students in the Canadian sample. The vast majority of the “Other (French)” students were from Quebec.

Figure 2 displays the socioeconomic gradients for these provinces across the six domains of achievement. The mathematics scores in each domain were scaled to a years-of-schooling metric corresponding to grade levels. Socioeconomic status is a statistical composite based on the level of education of the students’ parents and the availability of various educationally-related possessions in the home. This variable was scaled to have a mean of zero and a standard deviation of one for the full Canadian sample.

![Figure 2. Relationship Between Mathematics Scores and Socioeconomic Status, by Achievement Domain](image)

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The findings indicate that the variation among provinces is fairly consistent across domains of mathematics achievement. Generally, Ontario had the lowest adjusted scores, while Quebec had the highest adjusted scores. The gap between these two provinces, for a student with a nationally average socioeconomic background, was more than a full year of schooling. The scores for the other provinces were similar to those of Ontario, although Alberta and B.C. were slightly higher in some domains. We are currently conducting further research using data from the TIMSS to discern why Quebec students have better results. Two plausible hypotheses are that Quebec has a more centralized and faster-paced curriculum, or that there are cultural differences in the amount of emphasis placed on mathematics.

The socioeconomic gradients were somewhat steeper for proportionality, fractions, and statistics, but generally the gradients were similar across domains of achievement. This is especially relevant to teachers, because the conceptual approach to teaching and learning particular domains of mathematics differs, and much of the discussion regarding the relatively low performance of students concerns domain-specific achievement.

Differences between the Sexes

Figure 3 displays differences between males and females across the ten provinces. Differences between the sexes were not large in algebra, geometry, fractions, and statistics. However, the disparities were very large in proportionality, with females...
scoring about one year of schooling lower than males. Females also scored about six months of schooling lower in the measurement domain. We do not have a good explanation as to why females scored lower in these domains.

In a separate study, Ma and Willms (1997) examined the extent to which American youth participated in advanced mathematics courses from grades 7 to 12. The participation rates for females were comparable to those of males through to the end of grade 11. However, in the final year of high school, a disproportionate number of females dropped out of advanced mathematics, such that they were less prepared for college and university courses. For many students, both males and females, it is a lack of desire to pursue advanced courses in mathematics, rather than a lack of cognitive ability, that leads them to drop out of mathematics courses in the last year of high school.

A Call for Stronger Monitoring Systems

This research indicates that there are large, statistically significant differences among the Canadian provinces in their mathematics achievement. The disparities between Quebec and the other provinces are particularly striking. The differences are evident across studies, and are not simply due to the socioeconomic background of the students. However, each of these studies has strengths and limitations. One of the strengths of the NLSCY is that the data include a number of measures of family background, such as household income, family structure, parents’ education and occupation. The IALS is also strong in this respect. However, these studies were not designed as studies of school effectiveness, per se, and thus do not have the best sampling design for determining why some schools do better than others. The TIMSS is somewhat better in this regard, but does not include strong measures of family background. Moreover, none of these studies collect comprehensive data on schooling processes, such as the disciplinary climate of the classroom, or parents’ involvement in their child’s schooling.

Thus, the immediate implication of this research is that we need stronger systems for monitoring schooling outcomes at the national and provincial levels. We need systems that are capable of assessing socioeconomic gradients, not only at the provincial level, but also at the levels of schools and wider communities. Our monitoring systems need to be able to shed light on why some provinces and schools are performing better than others, and where efforts at school reform should be directed.

References:


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The aims of the Atlantic Centre for Policy Research are to conduct policy research that will help Canadian communities provide better education and care for their children, to contribute to the training of social scientists in the areas of statistics and research methods, and to contribute to capacity-building efforts in developing countries.