University of New Brunswick

Department of Mathematics & Statistics

and

Department of Mathematical Sciences

June 9, 2016 6 - 9 p.m.

Calculus Challenge Exam

Last name:			
First name:			
Middle initial(s):			
Date of birth:	year:	month:	day:
High school:			
Teacher:			
Credit at:	UNB-Fredericton	UNB-Saint John	

Instructions

1. Total points: 100.
2. Write all solutions on the paper provided.
3. Show your work! Full marks are awarded only when the answer is correct and supported with reasons.
4. Simplify answers unless directed otherwise.
5. Calculators and other electronic devices are forbidden.
6. Notes, books, scrap paper and aids of any kind are forbidden.
7. Good luck!

MARKS

1. Find y'. Answers **need not** be simplified.

[4] (a) $y = \left(x^2 + \frac{1}{4x}\right)^3$

[4] (b) $y = \arctan(x^2) - e^{\sin(x)}$

[4] (c) $y = \cos^2(t) + 4\log_2(t)$

[4] (d) $y = x^{-5} + 5^x - \frac{5}{x}$

[5] (e) $y = (1+x)^{\frac{1}{x}}$

2. Find the indicated limits. If the limit is infinite, write "$+\infty$" or "$-\infty$". If the limit does not exist and is not infinite, write "DNE". You may use L'Hospital's Rule where appropriate. Justify your answers.

[3] (a) $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$

[3] (b) $\lim_{x \to \infty} \dfrac{\ln(x)}{\sqrt{x}}$

[4] (c) $\lim_{x \to 0^+} \left(e^{-x} + x\right)^x$

[3] (d) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 + 1}}{2x + 3}$

3. Answer (a) and (b) for the function f defined below.

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}.$$

[3] (a) Find $\lim_{x \to 1} f(x)$.

[2] (b) Is f continuous at $x = 1$? Justify your answer.

[6] 4. Use the limit definition of the derivative to compute $f'(3)$ where $f(x) = \frac{5}{x+2}$.

5. Consider the curve described by the equation

$$x^4 + x^2y^2 - y^2 = 0.$$

[5] (a) Find $\dfrac{dy}{dx}$ (in terms of x and y).

[3] (b) Find an equation of the line tangent to the curve at the point $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

6. A particle is moving with velocity at time t given by

$$v(t) = t^2 - 3\sqrt{t} \quad \text{m/s}.$$

The position of the particle at $t = 4$s is 8m (that is, $s(4) = 8$).

[2] (a) What is the acceleration at time t?

[5] (b) What is the position at time t?

[2] 7. (a) State the Mean Value Theorem.

[3] (b) Illustrate the theorem using the function $f(x) = x^3 + 5x$ on the interval $[0, 2]$.

[3] (c) Explain why the Mean Value Theorem appears to fail for the function $g(x) = |x - 3|$ on the interval $[0, 16]$.

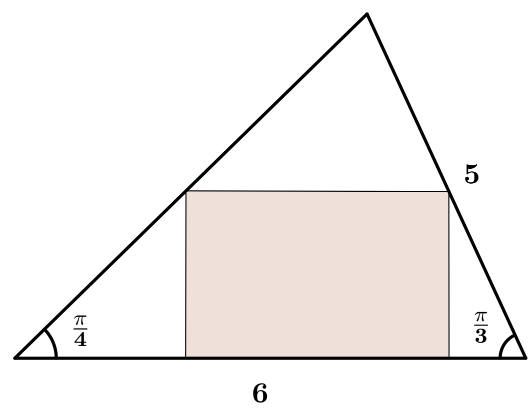
8. Consider the function $f(x) = \sqrt{1 - e^x}$.

[3] (a) Let $g(x) = f^{-1}(x)$, that is, g is the inverse function of f. Find and simplify $g(x)$.

[2] (b) Find and simplify $g'(x)$.

[2] (c) Find and simplify $f'(g(x))$.

[8] 9. Find the area of the largest rectangle that can be inscribed in a triangle with angles $\pi/4$ and $\pi/3$ and side lengths 5 and 6, as pictured below.



10. This question extends over two pages. Answer all parts of this question with regard to the graph of $y = f(x)$, where $f(x) = \dfrac{x+5}{(x+3)^2}$

[1] (a) State the domain of f.

[4] (b) Find all vertical and horizontal asymptotes of the graph.

[4] (c) Find the intervals on which f is increasing, and the intervals on which it is decreasing.

[1] (d) Find the coordinates of any local/relative extrema.

[4] (e) Find the intervals on which the graph is concave up, and the intervals on which it is concave down.

[1] (f) Find the coordinates of any points of inflection.

[2] (g) Sketch the graph of $y = f(x)$, taking care to incorporate your answers to (a) through (f) above.

(100)