University of New Brunswick
Department of Mathematics & Statistics
and
Department of Mathematical Sciences

June 3, 2015 6 - 9 p.m.

Calculus Challenge Exam

Last name:			
First name:			
Middle initial(s):			
Date of birth:	year:	month:	day:
High school:			
Teacher:			
Credit at:	UNB-Fredericton	UNB-Saint John	

Instructions

1. Total points: 100.
2. Write all solutions on the paper provided.
3. Show your work! Full marks are awarded only when the answer is correct and supported with reasons.
4. Simplify answers unless directed otherwise.
5. Calculators and other electronic devices are forbidden.
6. Notes, books, scrap paper and aids of any kind are forbidden.
7. Good luck!

MARKS

1. Find y'. Answers **need not** be simplified.

[4] (a) $y = x^2 + 2^x + e$

[4] (b) $y = \dfrac{\pi - x^{-1}}{\pi + \sec(2x)}$

[4] (c) $y = \sqrt{\cos(t) - \arctan(2t)}$

[4] (d) $y = e^{x^2 - x} - 8\log_3(x)$

[4] (e) $y = t^2 \sin^2(t)$

[4] (f) $y = (1 - x)^{\sqrt{x}}$

[6] 2. Find the absolute maximum value and the absolute minimum value of $\dfrac{\ln x}{x^3}$ on the interval $[1, e]$.

[5] 3. (a) Use the limit definition of the derivative to compute the derivative of $f(x) = \frac{4}{3+x}$.

[3] (b) Find an equation of the tangent line to the curve given by $y = f(x) = \frac{4}{3+x}$ at $x = 1$.

4. Find the indicated limits. If the limit is infinite, write "$+\infty$" or "$-\infty$". If the limit does not exist and is not infinite, write "DNE". You may use L'Hospital's Rule where appropriate. Justify your answers.

[3] (a) $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$

[3] (b) $\lim_{x \to -\infty} \frac{4x+3}{\sqrt{4x^2+9}}$

[3] (c) $\lim_{z \to -4} \frac{\sqrt{z^2+9} - 5}{z+4}$

[3] (d) $\lim_{t \to \infty} te^{-t^2}$

[6] 5. Consider the curve described by the equation

$$\tan(x - y) = \frac{y}{1 + x^2}.$$

Find $\frac{dy}{dx}$ (in terms of x and y). No need to simplify your answer.

6. Answer (a) and (b) for the function f defined below.

$$f(x) = \begin{cases} \cos(x) & \text{if } x < 0 \\ d - x & \text{if } 0 \le x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

[3] (a) For what value of d would f be continuous on $(-\infty, 1]$?

[3] (b) Is f continuous on $(-\infty, \infty)$ for the value of d you found in part (a)? Justify your answer.

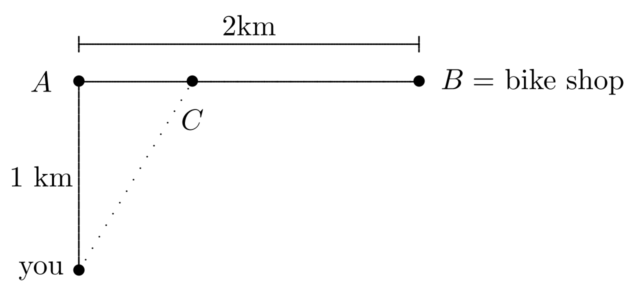
7. Consider the function $f(x) = \sqrt{3 - x}$.

[3] (a) Find the inverse function $f^{-1}(x)$.

[2] (b) Verify your result in part (a) by finding $(f^{-1} \circ f)(x)$.

[2] (c) Find the domain and range for $f^{-1}(x)$ that you found in part (a).

[8] 8. You are cycling on a trail, 1 kilometre from a road, when you get a flat tire. You have neglected to bring a repair kit, but luckily there is a bike shop 2 kilometres down the road. If your walking speed through the woods is 3 km/h and along the road is 5 km/h, what is the fastest way to get to the bike shop? Assume that you walk in a straight line through the woods until you hit the road at point C. (C may equal A or B or any point in between.)



9. This question extends over two pages. Answer all parts of this question with regard to the graph of $y = f(x)$, where $f(x) = 2 + \frac{1}{x} - \frac{1}{x^2}$.

[1] (a) State the domain of f.

[2] (b) Find any vertical or horizontal asymptotes in the graph.

[4] (c) Find the intervals on which f is increasing, and the intervals on which it is decreasing.

[1] (d) Find the coordinates of any local/relative extrema.

[4] (e) Find the intervals on which the graph is concave up, and the intervals on which it is concave down.

[1] (f) Find the coordinates of any points of inflection.

[2] (g) Sketch the graph of $y = f(x)$, taking care to incorporate your answers to (a) through (e) above.

10. A cylindrical water tank has a diameter of 6 metres and is 6 metres tall. The surface area of a closed cylinder is given by $2\pi rh + 2\pi r^2$ and the volume by $\pi r^2 h$, where r is the radius and h is the height.

[4] (a) Find the rate at which the water level in the tank is rising, if water is pumped into the tank at 5π m^3/min.

[4] (b) Another tank of the same dimensions is also being filled at a rate of 5π m^3/min, but it is observed that there is a small crack in the bottom of the tank. If the level of the water in the tank is rising at 0.5 m/min when the water is 4 metres deep, how fast is the tank leaking?