University of New Brunswick
Department of Mathematics & Statistics
and
Department of Mathematical Sciences

June 5, 2014 6 - 9 p.m.

Calculus Challenge Exam

Last name:	
First name:	
Middle initial(s):	
Date of birth:	year: month: day:
High school:	
Teacher:	

Instructions

1. Total points: 100.
2. Write all solutions on the paper provided.
3. Show your work! Full marks are awarded only when the answer is correct and supported with reasons.
4. Simplify answers unless directed otherwise.
5. Calculators and other electronic devices are forbidden.
6. Notes, books, scrap paper and aids of any kind are forbidden.
7. Good luck!

MARKS

1. Find y'. Answers **need not** be simplified.

[4] (a) $y = \left(\dfrac{1+x}{1-x}\right)^4$

[4] (b) $y = \arctan(3x) - 9\log_5(x)$

[4] (c) $y = \sin^2(t) + 2^{\sqrt{t}}$

[4] (d) $y = e^{\sinh(2t)}$

[6] (e) $y = (\cos(x))^{3x}$

2. Consider the curve described by the equation

$$2xy + \pi \sin(y) = 2\pi.$$

[5] (a) Find $\dfrac{dy}{dx}$ (in terms of x and y).

[3] (b) Find an equation of the line tangent to the curve at the point $\left(1, \frac{\pi}{2}\right)$.

3. Answer (a) and (b) for the function f defined below.

$$f(x) = \begin{cases} \cos(x) & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$$

[3] (a) Find $\lim_{x \to 0} f(x)$.

[3] (b) Is $f(x)$ continuous at $x = 0$? Justify your answer.

4. Jessica has a thermometer outside her window which records a temperature of 19°C at 1pm and 4°C at 11pm the same day. For each of the two statements below, do three things: (i) Use a theorem from calculus to argue that the statement is true, OR to argue that the statement is false; (ii) name the theorem you are using; (iii) list any assumptions you must make to apply the theorem.

[3] (a) At some point between 1pm and 11pm, the temperature was 7°C.

[3] (b) At some point between 1pm and 11pm, the temperature was dropping by 1.5 degrees per hour.

[7] 5. Use the limit definition of the derivative to compute the derivative of $f(x) = \sqrt{x-2}$.

6. Find the indicated limits. If the limit is infinite, write "$+\infty$" or "$-\infty$". If the limit does not exist and is not infinite, write "DNE". You may use L'Hospital's Rule where appropriate. Justify your answers.

[3] (a) $\lim_{x \to 1} e^{x^2 - x}$

[3] (b) $\lim_{t \to -2} \frac{t+2}{t^3+8}$

[3] (c) $\lim_{x \to -\infty} \frac{\sqrt{9x^2 - x}}{x+1}$

[3] (d) $\lim_{z \to 0} \left(\frac{1}{z\sqrt{1+z}} - \frac{1}{z} \right)$

7. A rock is thrown upwards on Mars. The velocity at time t is given by

$$v(t) = 18 - 1.8t \ \text{ m/s.}$$

[3] (a) What is the acceleration at time t?

[3] (b) What is the height at time t? (Assume the height at $t = 0$ is 0).

[3] (c) What is the maximum height the rock reaches?

[8] 8. A shipping company requires rectangular packages shipped within Canada to have length plus girth no greater than 3m, where length is the length of the longest size, and girth is the distance around – that is, the perimeter of a cross section. Find the dimensions of the package with the largest volume that can be shipped, assuming it has a square cross section.

9. This question extends over two pages. Answer all parts of this question with regard to the graph of $y = f(x)$, where

$$f(x) = \left(\frac{2+x}{2-x}\right)^3, \qquad f'(x) = \frac{12(2+x)^2}{(2-x)^4}, \qquad f''(x) = \frac{24(2+x)(6+x)}{(2-x)^5}.$$

[1] (a) State the domain of f.

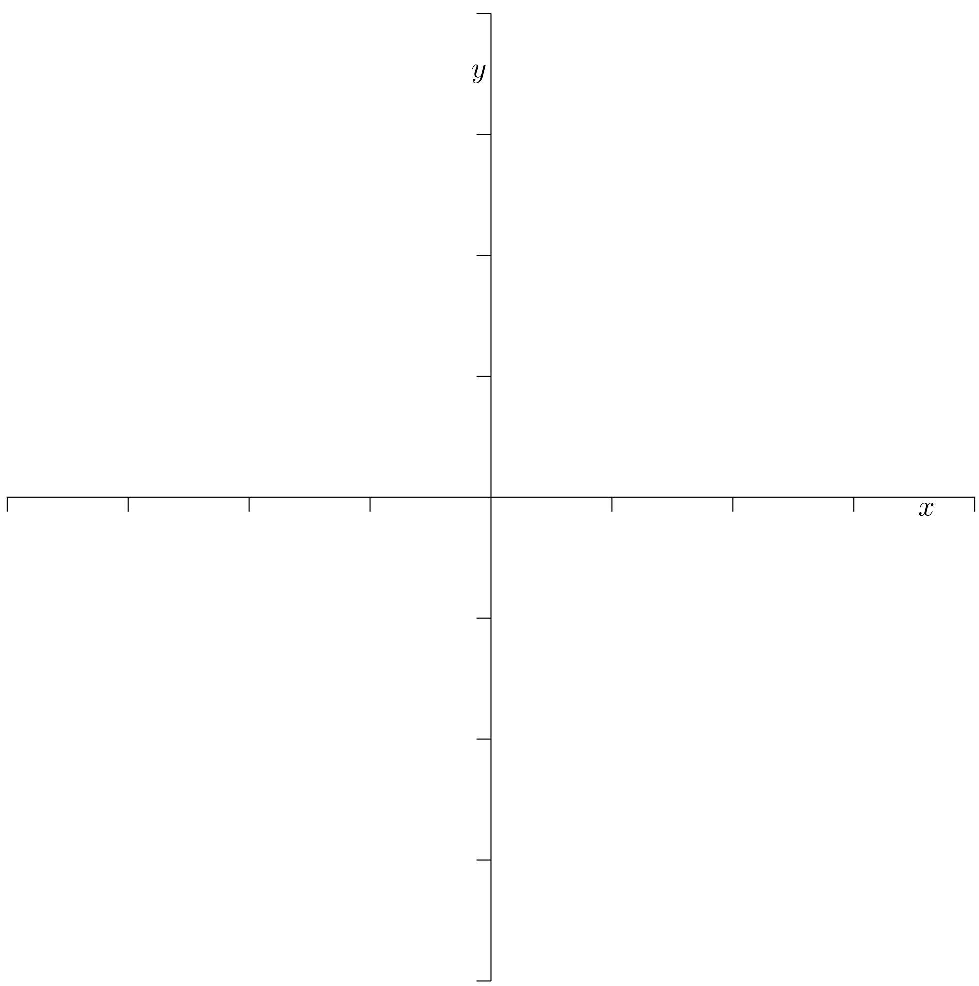
[2] (b) Find the intervals on which f is increasing, and the intervals on which it is decreasing.

[4] (c) Find any vertical or horizontal asymptotes in the graph. Use limits to justify your answers.

[4] (d) Find the intervals on which the graph is concave up, and the intervals on which it is concave down.

[2] (e) Find the coordinates of any points of inflection.

[2] (f) Sketch the graph of $y = f(x)$, taking care to incorporate your answers to (a) through (e) above.



[7] 10. A tank in the shape of a cone, 4m deep and 2m in diameter at the top, is being filled with water at a rate of 3 $\mathrm{m}^3/\mathrm{min}$. At what rate is the water level rising when the tank holds $\pi/6$ m^3 of water? (The volume of a cone with base radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(100)