University of New Brunswick
Department of Mathematics & Statistics
and
Department of Mathematical Sciences

June 6, 2013 6 - 9 p.m.

Calculus Challenge Exam

Last name:			
First name:			
Middle initial(s):			
Date of birth:	year:	month:	day:
High school:			
Teacher:			

Instructions

1. Total points: 100.
2. Write all solutions on the paper provided.
3. Show your work! Full marks are awarded only when the answer is correct and supported with reasons.
4. Simplify answers unless directed otherwise.
5. Calculators and other electronic devices are forbidden.
6. Notes, books, scrap paper and aids of any kind are forbidden.
7. Good luck!

1. Find the derivative. Answers **need not** be simplified.

(a) $y = x \cdot 2^{5x-4}$ (4)

(b) $y = \dfrac{x^8 + 2}{5 + \cot x}$ (4)

(c) $y = \dfrac{1}{2} \tan^{-1}(2x)$ ($\tan^{-1}(2x)$ is the same as $\arctan(2x)$.) (4)

(d) $y = \ln(\sqrt{1 + 3x})$ (4)

(e) $y = \sin^2(e^{6x})$ (4)

2. Answer (a) and (b) with regard to the curve described by the equation below.

$$e^x y + y^2 = e^x$$

(a) Find $\dfrac{dy}{dx}$. (5)

(b) Are there any points on the curve at which the tangent line is horizontal? Justify your answer. (3)

3. Find $\dfrac{dy}{dx}$ for $y = (\sin x)^{\tan x}$. Express your answer as a function of x. (6)

4. Use the limit definition of the derivative to compute the derivative of $f(x) = \dfrac{1}{x-2}$. (6)

5. Find the indicated limits. If the limit is infinite, write “$+\infty$” or “$-\infty$”. If the limit does not exist and is not infinite, write “DNE”. You may use L’Hospital’s Rule where appropriate. Justify your answers.

(a) $\displaystyle\lim_{x\to 2} \frac{x^2 - 7x + 10}{x - 2}$ (3)

(b) $\displaystyle\lim_{x\to 0} \frac{5^x - 5}{3^x}$ (3)

(c) $\displaystyle\lim_{t\to 0} (1 + 2t)^{1/t}$ (3)

(d) $\displaystyle\lim_{x\to\infty} \frac{x}{e^{\sqrt{x}}}$ (3)

6. Find all vertical and horizontal asymptotes for $y = \dfrac{(2-x)^3}{x^3+x}$. Use limits to justify your answers. (6)

7. This question extends over two pages. Answer all parts of this question with regard to the graph of $y = f(x)$, where

$$f(x) = -1 + \frac{x^2}{2} + \frac{x^3}{3}$$

(a) State the domain of f. (1)

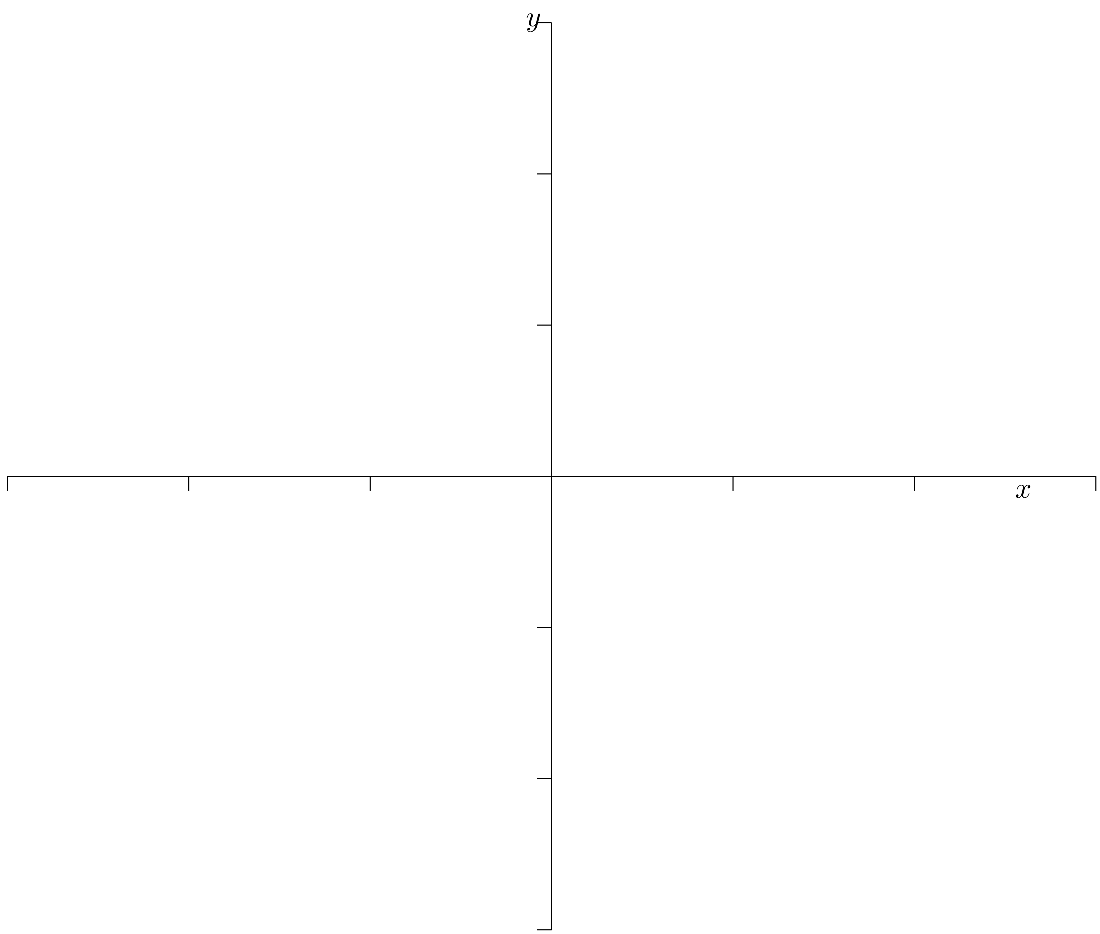
(b) Find the intervals on which f is increasing, and the intervals on which it is decreasing. (4)

(c) Find the coordinates of any local/relative extrema. (2)

(d) Find the intervals on which the graph is concave up, and the intervals on which it is concave down. (4)

(e) Find the coordinates of any points of inflection. (2)

(f) Sketch the graph of $y = f(x)$, taking care to incorporate your answers to (a) through (e) above. (2)



8. A jogger runs at a constant speed around a circular track of radius 20 m. Let (x, y) be her coordinates where the origin is the centre of the track. At what point(s) on the circle is her x-coordinate changing at the same rate as her y-coordinate? (7)

9. Find the general antiderivative of $f(x) = 2x + \sec^2 x - \sqrt{x} + 42$. (4)

10. We want to construct a small box whose base length is 3 times the base width. The material used to make the top and the bottom costs 10¢ per square centimetre and the material used to make the sides costs 6¢ per square centimetre. If the box must have a volume of 60 cubic centimetres, determine the dimensions that will minimize the cost of making the box. (7)

11. Consider the function

$$f(x) = \begin{cases} b + x, & x \le -1, \\ 1 - x, & -1 < x < 0, \\ \dfrac{1}{1+x^2}, & 0 \le x, \end{cases}$$

where b is a parameter.

(a) For what value(s) of b is f continuous at $x = -1$? (3)

(b) Find $\lim_{x \to 0} f(x)$ if it exists. Justify your answer. (3)

(c) Assuming $b = 4$, find the absolute maximum of f on $(-\infty, \infty)$. (3)