## Vita

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Arizona State University (2020)
Bachelors of Science (Honors)

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Masters of Science

# Kaplansky's Conjectures and Actions on CAT(-1) Spaces

UNIVERSITY OF NEW BRUNSWICK

THESIS DEFENCE AND EXAMINATION

in Partial Fulfillment

of the Requirement for the Degree of
Master of Science

by

**Matthew D. Brannock**

in the Department of Mathematics & Statistics

U.N.B., Fredericton, N.B.

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**12:30 p.m.**

Science Library, rm 108 & via MS TEAMS

Examining Committee

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| Dr. Nicholas Touikan | Supervisor |
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## Abstract

In the thesis we provide specific conditions on a ring $R$ and group $G$ under which the group ring $RG$ will satisfy the Kaplansky Conjectures on the existence or non-existence of non-trivial units, non-trivial zero-divisors and non-trivial idempotents in the group ring. We give a chain of implications on properties that a group must have to satisfy a these conjectures. Specifically, we define a special type of action of a group on a type of metric space of non-positive curvature called CAT(-1) space, and show that if a group acts in this special way on a CAT(-1) space, then this action will behave in a way that we call spherically diffuse. We then prove that if a group acts on a metric space in a spherically diffuse manner, then the group itself must be diffuse, which is a group theoretical property independent of any sort of metric space theory. Transitioning into group theoretical properties, we prove that if a group is diffuse then it satisfies something called the Unique Product Property. We then prove that if a group satisfies this property, then the group ring formed by this group and any integral domain will satisfy the Kaplansky Conjectures.