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Lee and Manhattan MWS and FWS Codes

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by

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in the Department of Mathematics & Statistics

U.N.B., Saint John, N.B.

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Examining Committee

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Abstract

Let q be a prime number and let $V = (F_q)^n$ be the vector space consisting of all the length n vectors whose components are the elements of the finite field F_q. We say that a subset C of V is a linear code if C is a subspace of V, namely for every two elements c_1 and c_2 in C and two scalars a_1 and a_2 in F_q, we have $a_1c_1 + a_2c_2$ is in C. The elements of C are called codewords. The finite field F_q which a code is over is called the alphabet. The space $(F_q)^n$ may be endowed with a weight function, that can be induced by a distance metric. Hamming weight, the weight function induced by the Hamming metric, is most widely used in coding theory. The size q of the alphabet, and the dimension k of a linear code, impose a maximum number of Hamming weights that a linear code can have, denoted $L_H(k,q)$. In [1], it was shown for any prime power q and any positive integer k that $L_H(k,q) = (q^k-1)/(q-1)$. Linear codes in which there are $(q^k-1)/(q-1)$ distinct Hamming weights were given the name maximum weight spectrum codes. In this work we determine that for any prime number q and any positive integer k the maximum number of distinct Lee weights that a linear code can have is $L_L(k,q) = (q^k-1)/2$ and the maximum number of distinct Manhattan weights that a linear codes can have is $L_M = q^k-1$. This is done by first identifying an upper bound on the functions $L_L(k,q)$ and $L_M(k.q)$ and then showing the bound is sharp by constructing codes which meet the bound with equality. We show that with respect to Hamming, Lee, and Manhattan weights there is a lower bound on the length of MWS codes of $n = (q^k-1)/(q-1)$. Linear codes in which there is at least one codeword of each weight observed in the ambient space are called full weight spectrum (FWS) codes. In [2] it was shown that the maximum length of a Hamming FWS code is 2^k-1. We show that with respect to Lee and Manhattan weights, there is an upper bound on the length of FWS codes. In particular, we show that the maximum length of a Lee FWS code is at least $n = [(q+1)/2)^k-1]/[(q+1)/2-1]$ and the maximum length of a Manhattan FWS code is $n = (q^k-1)/(q-1)$. We leave a generalized result for the maximum length of an FWS code with respect to a component wise metric as an open problem.