Module 1: Corporate Finance and the Role of Venture Capital Financing
Time Value of Money

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1.0 Interest – The Cost of Money

1.01 Introduction to Interest

Interest is the cost of having money available for use for a period of time. If you borrow $25,000 to buy a car and repay the loan over 4 years, you will pay back more than $25,000. The difference is the interest or cost of having use of the money to buy the car.

If you use your own money to buy the car instead of borrowing, you will not pay interest, but there is a cost of using your money for the purchase, i.e. the interest you would have earned if you had invested the $25,000.

Many business transactions involve interest – e.g. borrowing money, lending money, or investing money with the objective of earning a return on investment. In all cases, there are two important considerations:

1. the amount of money tied up
2. the time or duration the funds will be unavailable for other purposes.

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1.02 Time Value of Money Taxonomy

P = present sum of money at time zero or now.
PV = present value
F = future sum of money
FV = future value
F_n = future sum of money at time n
N = total number of periods
n = total number of periods to a future point in time
i = interest rate per period
I = total interest earned over N periods
t_0 = time zero or beginning of time horizon (now)
t_n = future point in time that is n periods away from t_0
A = a uniform series of payments

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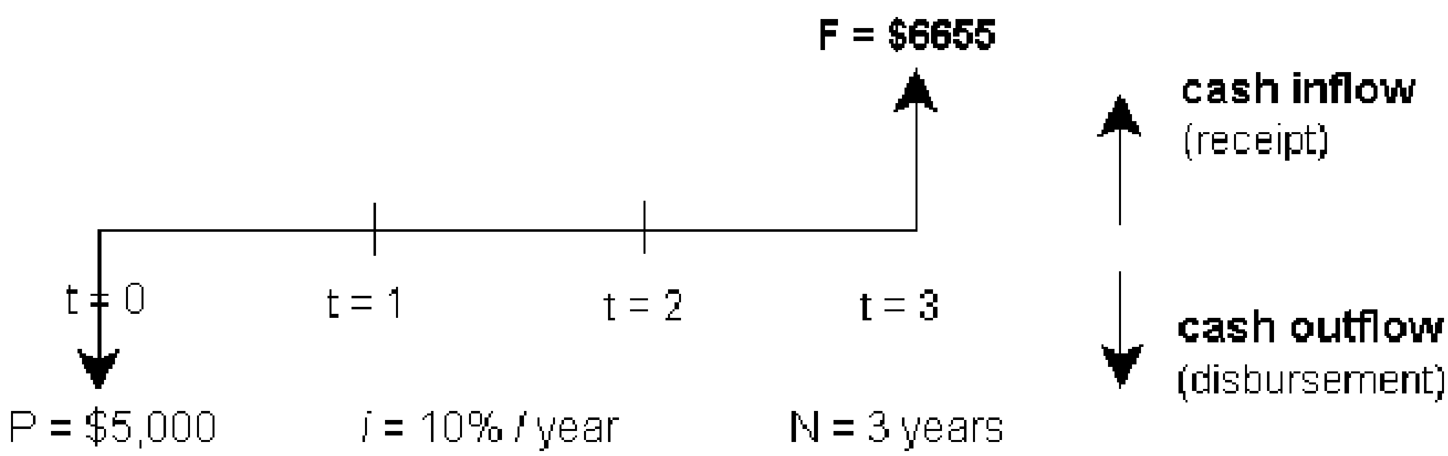
1.03 Cash Flow Diagrams

Cash flow diagrams are useful in economic analysis. They depict whether a cash flow is an inflow (receipt) or outflow (disbursement) and when each cash flow occurs.

The normal convention is an upward pointing arrow for a receipt and a downward pointing arrow for a disbursement. Time zero is at the extreme left of the diagram and future periods are to the right of time zero.

Cash Flow Diagram Example



Cash Flow Diagram Exercise

Create a cash flow diagram for the following cash flows:

1. initial investment of $275,000 at t=0;
2. annual operating costs of $15,000/year for 5 years (assume they occur at the end of the year);
3. a one time maintenance cost of $20,000 at t=3;
4. annual revenues of $75,000 (assume they occur at the end of the year);
5. a salvage value of $10,000 at t=5.

Answer on last page.

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1.1 Calculating Interest

1.11 Simple Interest

Most business transactions involve compound interest. However, it is useful to be familiar with simple interest if for no other reason than to understand **the power of compound interest**.

In simple interest, the interest expense for each period is determined by multiplying the interest rate by the principal outstanding at the beginning of the period.

If **I = total interest earned**, then **I = (iP)N**

Example
How much will a $100,000 investment grow to in 3 years at a simple interest rate of 15% per year? With simple interest the interest rate is multiplied by the investment amount each period to determine the interest earned for the period. In a borrowing situation, the interest rate is multiplied by the principal outstanding at the beginning of each period to determine the interest charge for the period. In this example, the interest earned each year is 15% times $100,000 or $15,000. Over 3 years, 3 times $15,000 or $45,000 will be earned. So the $100,000 investment will grow to $145,000 over 3 years at a simple interest rate of 15%. **I =(iP)N** $= (0.15 \text{ x } \$100{,}000)^3$ **= $45,000**

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1.12 Compound Interest

With compound interest, the interest earned or charged (investment or loan) each period is added to the investment amount or principal outstanding. This new sum is used to determine interest earned or charged for the next period.

At the end of period 1:

F1 = **P + Pi or**

F1 = **P(1+i)**

At the end of period 2:

F2 = **[(P+Pi) + (P+Pi)I] or**

F2 = **[P(1+i) + P(1+i)I] or**

F2 = **[P(1+i)((1+i)] or**

F2 = **[P(1+I)^n]**

Example
How much will a $100,000 investment grow to in 3 years at a compound interest rate of 15% per year? With compound interest the interest rate is multiplied by the investment amount each period to determine the interest earned for the period. The interest is added to the investment amount and this sum then becomes the investment amount on which to determine interest earned for the next period. In a borrowing situation, the interest rate is multiplied by the principal outstanding at the beginning of each period to determine the interest charge for the period. The interest is added to the principal and this sum then becomes the new principal on which to determine interest charged for the next period. In this example, the interest earned in the first year is 15% times $100,000 or $15,000. This is added to the $100,000 to get an investment amount of $115,000 for the second year. The interest earned in the second year will be 15% time $115,000 or $17,250. This is added to the $115,000 to get an investment amount of $132,250 for the third year. The interest earned in the third year will be 15% times $132,250 or $19837.50 giving an investment amount of $152,087.50. **If $F_3 = P(1+i)^n$, then $F_3 = \$100,000(1+0.15)^3 = \$152,087.50$** So the $100,000 investment will grow to $152,087.50 over 3 years at a compound interest rate of 15%. This compares with $145,000 at simple interest. So you should now understand the power of compound interest!

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1.2 The Concept of Economic Equivalence

1.21 Introduction

Because of the time value of money and the power of compound interest, we know that receiving $100,000 today is not the same as receiving $100,000 three years from today if there is an opportunity to earn interest (say 15% compound interest) on $100,000 received today.

You have already found out that if you receive $100,000 today and invest it at 15% compound interest it will grow to $152,087.50 over 3 years. Hence $100,000 received today is **equivalent** to receiving $152,087.50 three years from today if there is an opportunity to invest the $100,000 at 15% compound interest.

This is the concept of **economic equivalence** i.e. cash flows of differing magnitude occurring at different times can be equivalent in value because of the time value of money.

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1.22 Equivalence with Multiple Cash Flows

The concept of economic equivalence is not limited to single payment cash flows e.g. $100,000 today being equivalent to $152,087.50 three years from now if the $100,000 can be invested at 15% compound interest.

Example

Your company has just borrowed money at an interest rate of 10% compounded annually. Your banker gives you the option of paying the loan off with 3 equal end of year payments of $17,500 (A) or one lump sum payment at the end of year 3 (B). What lump sum payment would be equivalent to the 3 end of year payments of $17,500?

Option A:

Option B:

Convert the payment the payment at t=1 to an equivalent payment at t=3.

F_3= $17,500$(1+0.1)^2$ = $21,180

Convert the payment the payment at t=2 to an equivalent payment at t=3.

F_3 = $17,500$(1+0.1)^1$ = $19,250

The third payment is at t=3 and does not need to be converted.

Therefore the equivalent lump sum payment at t=3 is $21,180 + $19,250 + $17,500 = $57,930.

Hence a single lump sum payment at t=3 of $57,930 is equivalent to 3 equal end of year payments of $17,500 from t=1 to t=3, given a compound annual interest rate of 10%.

Given these two choices by your banker, you would be indifferent from an economic perspective.

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1.3 Nominal and Effective Interest

Many financial products such as loans, bonds, credit cards, savings accounts, etc, feature compounding of interest more than once per year. The compounding frequency could be semi-annually, quarterly, monthly or even daily.

The annual interest rate before considering the effect of compounding more than once per year is known as **the nominal interest rate.** It is also often referred to as **the annual percentage rate (APR).**

When compounding frequency is more than once per year the amount of interest earned or charged will be greater than it would be with annual compounding. The higher annual rate of interest that results from more frequent compounding is known as **the effective interest rate.** Given a nominal annual interest rate and the frequency of compounding, the effective annual interest rate can be calculated from the formula: **$k = (1 + k / /q)^q - 1$** where:

k = the effective interest rate
k / = the nominal interest rate
q = the frequency of compounding per year

Example
What is the effective annual interest rate if the nominal annual interest rate is 12% and the interest is compounded monthly? In this example, k / = 0.12 and q =12 **$k = (1 + k / /q)^q - 1$** **$= (1 + 0.12/12)^{12} - 1$** **= 0.127 or 12.7%**

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1.31 Cash Flow Diagram Exercise Answer