Bilevel Programming for Evaluating Revenue Strategy of Railway Passenger Transport Under Multimodal Market Competition

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A bilevel programming approach is used to study the strategies of increasing revenue of the railway agency running between Beijing and Tianjin, China. Bilevel programming approaches have been used in many studies to tackle a variety of transportation problems, but rarely for railway revenue strategy analysis. In this paper, the upper-level problem of the bilevel programming is to determine optimal pricing, speed, and level-of-service (LOS) strategy that maximizes the revenue of the railway agency. The lower-level problem describes passengers' mode choice behavior under a transportation market with three competing modes: bus, rail, and car. The lower-level problem is to minimize the traveler's cost in terms of money, time, and other related factors such as comfort and safety. A generalized cost function, considering these factors together with a logit model, is used to simulate travelers' mode choice behavior. Study results clearly show that the bilevel programming is appropriate for the problem studied here. The results indicate that, to increase its revenue, the railway agency should focus on not only the pricing but also travel time and LOS. A pricing breakpoint of about ¥31 (¥7 = US$1) is found, as it results in the highest revenue for all traveling speeds and LOS. Further increase of the price leads to reduced revenue. A consistent revenue increase trend is observed for all higher traveling speeds and LOS, which emphasizes that the railway agency should pay attention to a combined revenue strategy.

Passenger transportation in China has experienced considerable development in recent years. However, rapid development results in higher-than-ever travel demand. Most passenger transport systems are working at capacity and suffer serious congestion during important holidays (i.e., Spring Festival and National Day). The experiences of developed countries demonstrate that it is important to establish a modern passenger transport system to achieve sustainable development. A fundamental requirement for such a system is the inclusion of various transportation modes that compete with each other. In this case, a mathematical model that can consider the characteristics of each mode and evaluate the benefit to individual passengers and carriers is essential to test proposed transportation strategies and policies.

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Transportation Research Record: Journal of the Transportation Research Board, No. 2117, Transportation Research Board of the National Academies, Washington, D.C., 2009, pp. 1–6.
DOI: 10.3141/2117-01

There has been growing interest in the development of integrated transport strategies in Europe, the United States, and elsewhere. Early work in the United Kingdom (*1*) suggests that system performance is likely to be particularly sensitive to the pricing elements, and that clever pricing and management measures should enable sustainable transport strategies to be developed. In contrast to the free market system, passenger transport market in China is not only governed by the law of supply and demand, but also restrained by past government interference, regulation, and subsidy. Previous studies (*2, 3*) have suggested that the integrated transport strategies for China should efficiently facilitate cooperation and competition among different modes. The aim of this paper is to develop a mathematical model using bilevel programming for testing different revenue strategies and policies to be implemented.

An understanding of travelers' behavior, especially mode choice, is important in evaluating revenue strategies of carriers. The mode choice model is an effective tool for evaluating the impact of transport strategies by estimating the shifts in mode shares. Mode shares, together with pricing information, largely determine the profit of carriers, and therefore, their competitiveness. McFadden (*4, 5*) proposed logit-based disaggregate models for which the user chooses one mode within a full set of choices that would provide him or her maximum utility. Oum and Gillen (*6*) developed an aggregate mode split model based on the user's behavior. More recently, Si and Gao (*7, 8*) introduced the Wardrop's user equilibrium principle into mode choice. They presented a stochastic user equilibrium (SUE) model that describes passengers' mode choice behavior under the market competition condition of several modes, while the congestion produced by excessive demand beyond the capacity is considered explicitly in the traveler's utility function. In this paper, a similar approach is used to model passenger mode choice and resulting carrier revenue.

A literature review indicates current research is limited to studying the feasibility of new railway transportation modes, such as high-speed (*9*) and magnetic levitation (*10, 11*). These studies also fall short with a "one-way analysis" in that a constant share of the new mode is assumed and then benefit–cost ratios for every mode being considered are computed and compared. In general, there is no interaction between the mode choice behaviors of travelers and revenue strategies of carriers. Therefore, it is not possible to test revenue policy with those frameworks.

In recent years, the bilevel programming approach has emerged as an important tool for tackling a variety of complicated transportation problems (*12–14*). In this paper, bilevel programming is used to test various strategies that may be used by the railway agency (Beijing Railway Bureau) to increase its revenue. The approach considers the behaviors of the transportation agency and passengers simultaneously. The upper-level problem is to determine an optimal strategy (e.g., a lower fare or higher operating speed) to maximize the revenue of the carrier. The lower-level problem is to simulate passengers' behavior with a mode choice model under a market condition influenced by the revenue strategy defined in the upper-level problem. The objective of the lower-level problem is to minimize passengers' generalized travel cost.

STOCHASTIC USER EQUILIBRIUM MODEL FOR MULTIMODAL PASSENGER TRANSPORTATION

To simplify illustration and computation of the proposed bilevel programming optimization model, it is assumed that the total travel demand from city r to city s, represented by q_{rs}, is given and constant during the period being considered. There are K transportation modes that passengers can choose from. Let f_k^{rs} represent the passenger demand of mode k ($k \in K$) between r and s. The fundamental relationship of this mode choice problem can be written as follows:

$$f_k^{rs} = q_{rs} P_k^{rs} \qquad \forall k, r, s \tag{1}$$

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall r, s \tag{2}$$

where P_k^{rs} is the probability of choosing mode $k \in K$ for all passengers.

Assume that the perceived generalized cost of each mode is a random variable consisting of the observed component and a perception error, which can be written as

$$T_k^{rs}\left(f_k^{rs}\right) = t_k^{rs}\left(f_k^{rs}\right) + \epsilon_k \qquad \forall k, r, s \tag{3}$$

where

$T_k^{rs}(f_k^{rs})$ = perceived cost of mode k ($k \in K$) between r and s,
ϵ_k = error term, and
$t_k^{rs}(f_k^{rs})$ = observed generalized cost of mode k ($k \in K$), which, in general, can be linearly formulated as

$$t_k^{rs}\left(f_k^{rs}\right) = \sum_n a_k^{(n)} w_k^{rs(n)}\left(f_k^{rs}\right) \qquad \forall k, r, s \tag{4}$$

where $w_k^{rs(n)}(f_k^{rs})$ represents the cost contributed by attribute n (e.g., price, travel time, convenience, comfort, and so on) of mode k ($k \in K$) between r and s, f_k^{rs}; $a_k^{(n)}$ is a weight parameter that reflects the sensitivity of the passengers to the corresponding attributes of a given transportation service or mode.

Based on a logit model, the probability of a given transportation mode k chosen by a passenger can be defined by calculating the probability that T_k^{rs} is lower than the perceived generalized cost of all other alternatives:

$$P_k^{rs} = \Pr\left(T_k^{rs} \le T_l^{rs} \qquad \forall l \in K, l \ne k\right) \tag{5}$$

where T_l^{rs} is the perceived cost of l between r and s respectively.

In this paper, the lower-level mode choice problem is formulated by the following SUE model:

$$\min Z(\mathbf{f}) = \sum_{rs}\sum_k f_k^{rs} t_k^{rs}\left(f_k^{rs}\right) - \sum_{rs}\sum_k \int_0^{f_k^{rs}} t_k^{rs}(w)\,dw - \sum_{rs} q_{rs} E\left[\min_{k\in K}\left\{T_k^{rs}\right\}\middle|\mathbf{t}^{rs}(\mathbf{f})\right] \tag{6}$$

where $E\left[\min_{k\in K}\left\{T_k^{rs}\right\}\middle|\mathbf{t}^{rs}(\mathbf{f})\right]$ is the expected minimum cost from city r to city s. It is a concave function of t_k^{rs}. The marginal perceived travel cost between r and s is the partial derivative of $E\left[\min_{k\in K}\left\{T_k^{rs}\right\}\middle|\mathbf{t}^{rs}(\mathbf{f})\right]$ with respect to t_k^{rs}, and it can be shown that it is equal to the probability of transportation mode k being chosen, that is according to Sheffi (*15*):

$$P_k^{rs} = \frac{\partial E\left[\min_{k\in K}\left\{T_k^{rs}\right\}\middle|\mathbf{t}^{rs}(\mathbf{f})\right]}{\partial\left(t_k^{rs}\right)} \qquad \forall k, r, s \tag{7}$$

REVENUE STRATEGY EVALUATION MODEL FOR RAILWAY AGENCY

It can be seen that users' mode choice behavior under multimodal market competition can be formulated by the mathematical programming model (Equation 6). The solutions to the model will provide the passenger demands for different transport modes with a given set of attributes (such as travel time, pricing, safety, comfort, and so on). Global evaluation of the likely effects of railway operating strategies thus becomes possible if it is assumed that the presence and attributes of any other transport modes (bus and car) are given and fixed. This assumption is deemed reasonable, especially for the short time period being considered, over which the railway revenue strategy is evaluated and adjusted. The model can now be formulated to search for an optimal operating strategy for the railway company such that a particular system performance criterion or system objective function is optimized. Many alternative system performance measures could be adopted as the optimal railway operating strategy. Meaningful objectives can be the market share of railway, revenue of the railway, or net revenue of the railway.

Because the central government in general pays for the infrastructure development in China, the objective function of a railway passenger transportation company in this case considers only factors influencing its revenue (although a more complicated objective function can be specified to consider both cost and revenue under a free market scenario in the United States and many other countries). In this paper, the objective function is formulated as the following:

$$F = \sum_{rs} f_k^{rs} u_k^{rs} \qquad k \in K \tag{8}$$

where u_k^{rs} denotes the average price of transport mode k ($k \in K$) between r and s.

In China, the government regulates transportation services with a specified pricing range. According to the current policies, the price of a given transportation service should fit into a lower and upper bound:

$$u_k^{rs(\min)} u_k^{rs} \le u_k^{rs(\max)} \qquad \forall k, r, s \tag{9}$$

where $u_k^{rs(\min)}$ and $u_k^{rs(\max)}$ represent the lower and the upper price boundary of mode k ($k \in K$) between r and s, as determined by the government.

The optimal revenue strategy of railway passenger transport under multimodal market competition can be represented as a leader–follower game in which the railway bureau is the leader, and the passengers who choose modes to travel are the followers. It is reasonable to assume that the railway bureau can influence a traveler's choice by changing the railway operating strategies, such as fare and traveling speed. The passengers make their mode choice decisions in a user-optimal manner according to the market situation. This interaction game can be represented by the following bilevel programming problem (U = upper-level programming and L = lower-level programming):

$$(\mathrm{U}) \quad \max F = \sum_{rs} f_k^{rs}\left(\mathbf{u}^{rs}\right) \cdot u_k^{rs} \tag{10a}$$

$$\text{s.t.} \quad u_k^{rs(\min)} \le u_k^{rs} \le u_k^{rs(\max)} \qquad \forall k, r, s \tag{10b}$$

where $f_k^{rs}(\mathbf{u}^{rs})$, usually called the response function, is implicitly defined by

$$(\mathrm{L}) \quad \min Z(f) = \sum_{rs}\sum_{k} f_k^{rs} t_k^{rs}\left(f_k^{rs}\right) - \sum_{rs}\sum_{k}\int_0^{f_k^{rs}} t_k^{rs}(w)\,dw - \sum_{rs} q_{rs} E\left[\min_{k\in K}\left\{T_k^{rs}\right\} \middle| \mathbf{t}^{rs}(\mathbf{f})\right] \tag{11}$$

The upper-level problem is to determine optimal pricing for a given passenger transport mode k ($k \in K$) to maximize its revenue, in the range of pricing regulated by the government. The lower-level problem is to develop a multimodal network SUE model, which simulates passengers' mode choice behavior under the competition of different modes. The objective function of the lower-level problem is to minimize passengers' perceived travel cost.

SENSITIVITY ANALYSIS–BASED SOLUTION ALGORITHM

Because of its intricate nature, bilevel programming has been identified as one of the most challenging yet useful tools for transportation problems. In the past, researchers have developed various solution algorithms. Abdulaal and LeBlanc (*12*) applied the Hook–Jeeves heuristic algorithm for a direct search of the solution for the network design problem. Fisk (*16*) developed an alternative single-level optimization model using a gap function to solve a signal control problem. Suwansirikul et al. (*17*) developed an alternative heuristic method, referred to as the equilibrium decomposed optimization algorithm, by approximating the derivative of the objective function of the upper-level problem. The sensitivity analysis method proposed by Tobin and Friesz (*18*), which calculates the derivatives of the equilibrium arc flows with respect to perturbation parameters, has been widely used for network equilibrium problems. Friesz et al. (*19*) applied it to solve network design problems. Yang and Yagar (*20*) used the same method for the inflow control problem on freeways. Yang (*21*) also used it for solving the queuing equilibrium network assignment problem, for which the derivatives of equilibrium link flows and equilibrium queuing time with respect to traffic control parameters are derived. Stephen and David (*22*) proposed an efficient computational method for the sensitivity analysis of probit-based SUE model applied to general networks. Their approach uses information of SUE path flows, but it is not specific to any particular equilibrium solution algorithm. Chan and Lam (*23*) used the sensitivity analysis-based algorithm to solve the bilevel programming model applied to road networks, with travel time information provided by the route guidance system.

Because of its popularity, the sensitivity analysis-based solution (*18–23*) is used in this study. The derivative of equilibrium demand of each mode is used to study the perturbation of the cost function. In this paper, the perturbation is assumed related to the prices of different modes only, and all the other attributes, such as travel time, comfort, convenience, and safety, are assumed constant. With the derivative information, the objective function and constraints of upper-level problem, which are basically a nonlinear function of the price of a given transport service, are linearly approximated using Taylor's formula, as follows. The first-order Taylor series expansion of $f_k^{rs}(\mathbf{u}^{rs})$ in the neighborhood of $\mathbf{u}^{rs} = \mathbf{u}^{rs(0)}$ can be written as

$$f_k^{rs}\left(\mathbf{u}^{rs}\right) \approx f_k^{rs}\left(\mathbf{u}^{rs(0)}\right) + \left.\frac{\partial \mathbf{f}^{rs}}{\partial \mathbf{u}^{rs}}\right|_{\mathbf{u}^{rs}=\mathbf{u}^{rs(0)}} \left(\mathbf{u}^{rs} - \mathbf{u}^{rs(0)}\right) \qquad \forall k, r, s \tag{12}$$

where the derivative terms are the Jacobian matrices of $\mathbf{f}^{rs}$ with respect to $\mathbf{u}^{rs}$, evaluated at $\mathbf{u}^{rs} = \mathbf{u}^{rs(0)}$, which can be calculated as follows (*23*):

$$\left.\frac{\partial \mathbf{f}^{rs}}{\partial \mathbf{u}^{rs}}\right|_{\mathbf{u}^{rs}=\mathbf{u}^{rs(0)}} = \left[\nabla^2 Z(\mathbf{f})\right]^{-1} \cdot \left[-\left(\nabla^2_{\mathbf{fu}} Z(\mathbf{f})\right)^T\right] \tag{13}$$

$$\nabla^2 Z(\mathbf{f}) = \sum_{rs} q_{rs}\left[\left(\nabla_{\mathbf{f}}\mathbf{t}\right)\cdot\left(-\nabla_{\mathbf{c}}\mathbf{P}^{rs}\right)\cdot\left(\nabla_{\mathbf{f}}\mathbf{t}\right)^T\right] + \nabla_{\mathbf{f}}\mathbf{t} + \nabla^2_{\mathbf{f}}\mathbf{t}\cdot\mathbf{R} \tag{14}$$

$$\nabla^2_{\mathbf{fu}} Z(\mathbf{f}) = \sum_{rs} q_{rs}\left[\left(\nabla_{\mathbf{f}}\mathbf{t}\right)\cdot\left(-\nabla_{\mathbf{c}}\mathbf{P}^{rs}\right)\cdot\left(\nabla_{\mathbf{u}}\mathbf{c}^{rs}(\mathbf{u})\right)\right] \tag{15}$$

The following matrix notations are used in the above equations:

$\nabla_{\mathbf{f}}\mathbf{t}$ = Jacobian matrix of the generalized cost vector,
$\nabla_{\mathbf{c}}\mathbf{P}^{rs}$ = Jacobian matrix of the mode choice probability vector for origin–destination pair r–s, and
$\mathbf{c}^{rs}(\mathbf{u})$ = relationship between the vector of generalized costs and the vector of perturbations.

The sensitivity analysis-based solution algorithm uses an iterative process between the upper-level and lower-level problem to improve the solution accuracy. Successive average is used to constantly update the price value. A well-known simple method can then be used for solving the upper-level programming problem. A simulation algorithm can be used to solve the lower-level probit-based SUE problem. One of the advantages of the simulation approach is that the random errors can be any types of distribution.

The algorithm used in this paper is summarized as follows:

Step 0. Determine an initial set of transport service price patterns $\{u_k^{rs(0)}\}$, $\forall k, r, s$. Set $n = 0$.

Step 1. Solve the lower-level problem that is a probit-based SUE model and obtain a set of demand patterns $\{f_k^{rs(n)}\}$, $\forall k, r, s$.

Step 2. Find the response function $f_k^{rs}(\mathbf{u}^{rs})$, $\forall k, r, s$, which will be subsequently used by the sensitivity analysis method.

Step 3. Solve the nonlinear programming of the upper-level problem and obtain an auxiliary set of transport prices $\{v_k^{rs(n)}\}$, $\forall k, r, s$.

Step 4. Update.

$$u_k^{rs(n+1)} = u_k^{rs(n)} + \frac{1}{n+1}\left(v_k^{rs(n)} - u_k^{rs(n)}\right) \qquad \forall k, r, s \tag{16}$$

Step 5. If $\max\{|u_k^{rs(n+1)} - u_k^{rs(n)}|\} \le \delta$ or $n = M$, then stop. Otherwise, let $n = n + 1$ and go to Step 1. M is the maximum number of iterations, and δ is a stopping parameter with a small value.

STUDY RESULTS

In this section, the results for evaluating the revenue strategy of Beijing Railway Bureau with bilevel programming are described. The data used are from a report prepared by the railway agency (*24*). The parameters used are estimated from the observed behaviors of a passenger sample.

Tianjin is an important harbor city in Northeast China, 137 km from Beijing. There are three major surface transportation modes between the two cities: bus, train, and car. In this study, the major variables such as travel time (represented by τ_k), price (represented by u_k), and the other factor synthesizing convenience, comfort, and safety (represented by c_k), are used to formulate the generalized cost function of each mode. It is assumed that the error term of perceived generalized cost function is an independently and identically distributed Gumbel variable. Under this assumption, the logit model can be used to simulate the passenger mode choice (*15*). The generalized cost function $t_k(\mathbf{f})$ in this study is defined as

$$t_k(\mathbf{f}) = a_1\tau_k + a_2u_k + a_3c_k \qquad \forall k \tag{17}$$

The attributes, including the travel time (τ_k), price (u_k), and the synthesized service factor (c_k), of the three surface modes studied in this example are given below (*24*). The total demand for one-way passenger transport between Beijing and Tianjin was estimated to be 25,000 by the year 2000 (*24*).

Mode	τ_k/h	u_k/¥	c_k
Bus	1.5	25	5.993
Train	1.6	20	6.053
Car	1.2	35	6.154

The weights corresponding to travel time, price, and the other factor in the generalized cost function defined above are $a_1 = 30$, $a_2 = 4.75$, and $a_3 = 20$. The alternative specific constants are not included in the generalized cost function because the estimation results show reasonable accuracy. $\beta = 0.01$ is used as the dispersion parameter for the proposed logit model. Again, these values are estimated from the report supplied by Beijing Railway Bureau (*24*). The lower and the upper price boundaries are ¥15 and ¥35, respectively (¥7 = US$1). Assuming that the attribute values of these three modes and the total passenger demand from Beijing to Tianjin are constant, through the bilevel programming used in this paper, the optimal price for railway transport is found to be around ¥30, with maximum revenue of ¥234,492. The equilibrium demand for different modes is $f_1 = 10{,}091$(bus), $f_2 = 7{,}816$ (railway) and $f_3 = 7{,}091$ (car), respectively.

Figure 1 shows the solution curve for the objective function of the upper-level problem against the iteration number. It can be found that the convergence of the solution algorithm is obtained at about the 13th iteration. The convergence pattern implies that the algorithm used in this paper is relatively smooth and "well behaved."

Next, an examination of the revenue of the railway agency between Beijing and Tianjin by varying the price of railway and traveling speed is conducted. Figure 2 shows the revenue of the railway against its price at different traveling speeds (and therefore travel time), with an assumption that all of the other attributes of the three modes are constant. It can be found that the maximum revenue is nearly always achieved at the "optimal" price of about ¥30, which implies that a price higher or lower than this would lead to a decrease in revenue. It is also found that the corresponding revenue of railway will increase as the traveling speed increases. For example, at speeds of 70 km/h, 80 km/h, 90 km/h, 100 km/h, and 110 km/h, the optimal prices of railway are about ¥30 and the corresponding revenues of the railway are ¥215,599, ¥220,237, ¥224,932, ¥229,685, and ¥234,492, respectively. As the railway speed increases to 120 km/h, 130 km/h, 140 km/h, 150 km/h, and 160 km/h, the optimal prices of railway are increased to about ¥32 and the corresponding revenues of the railway are ¥239,404, ¥244,396, ¥249,443, ¥254,545, and ¥259,699 respectively. These observations can be explained by the fact that more and more passengers will choose the railway to travel if the railway can significantly save their travel time, with only a marginal increase of their financial burden.

Figure 3 shows the revenue of the railway against its price at the different levels of service (LOS) represented by c_k in Equation 17. Basically, Figure 3 shows a similar trend to that shown in Figure 2. It can be found, in general, that the revenue of the railway increases as the price increases and the LOS improves from a synthesized service factor of 5.0 to 6.2. The peak of the revenue appears to occur at the price of ¥32 and the service level of 6.2. It can be found that increasing the price after that point leads to declines in revenue.

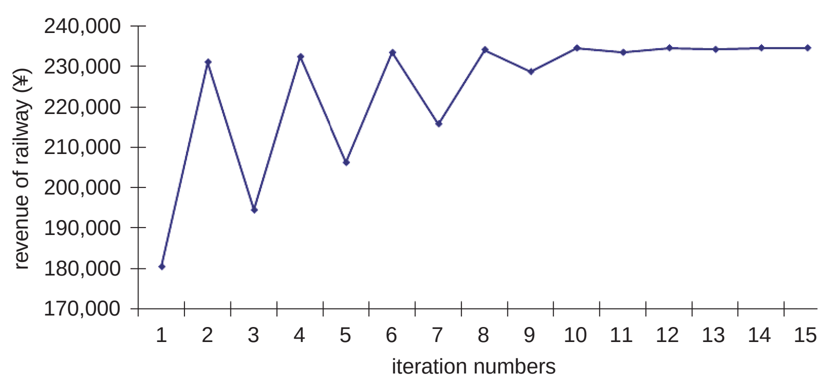

FIGURE 1 Convergence curve of algorithm.

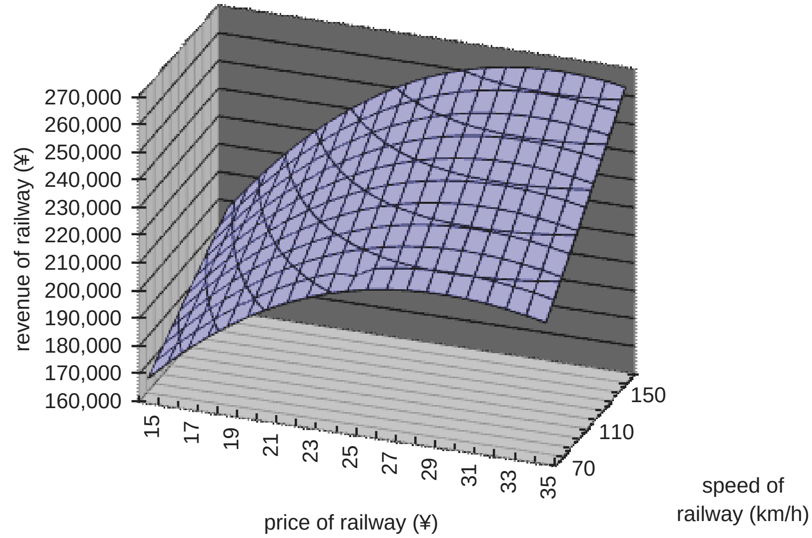

FIGURE 2 Revenue of railway against price at different speed levels.

These results clearly indicate that in the passenger transport market in China, especially in the areas where the competition between different modes is intensive, it will not be possible to increase the revenue of the railway by merely increasing the price. Figure 2 and 3 clearly show that the maximum revenue occurs at a price of about ¥30, and further increase of the price only leads to the decline in revenue. However, a higher traveling speed and LOS will help increase the revenue when the price is set at a given value. These observations from Figures 2 and 3 again emphasize that pricing strategy may be most effective only under a higher level of travel speed and service, and not by itself.

CONCLUSION

In this paper, a bilevel programming approach is used to search for an optimal revenue strategy for a railway agency in China. The approach studies the effects of different pricing and LOS policies on passengers' mode choice behavior in the Beijing and Tianjin corridor, where three major surface modes—bus, railway, and car—compete. The upper-level problem is to determine an optimal price that maximizes the revenue of the railway agency, although its value is subject to a range regulated by the government. The lower-level problem represents a multimodal network equilibrium model that describes passenger's mode choice behavior under a free market competition condition and its objective is to minimize passenger's perceived travel cost.

Study results clearly illustrate the usefulness of the approach proposed in this paper. The bilevel programming is shown to be able to tackle complicated transportation problems by splitting them into two independent yet connected parts (i.e., upper- and lower-level problems). A set of constraints, such as pricing range and utility maximization, can be easily included in the model. Study results from this paper show that the railway agency should not use pricing alone, but a combination of pricing with higher operating speed and LOS, to maximize its revenue.

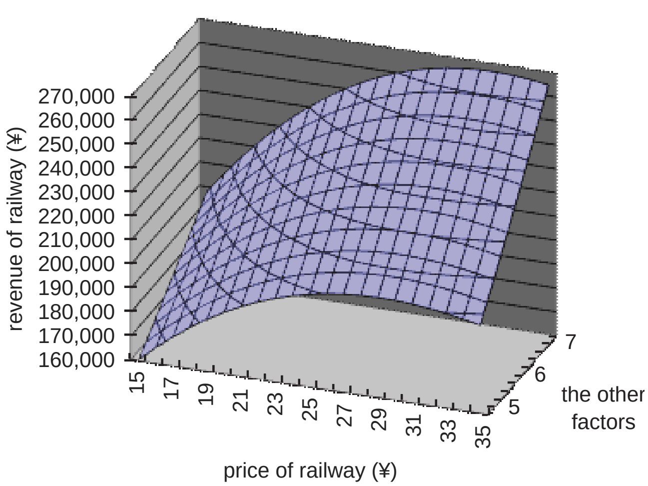

FIGURE 3 Revenue of railway against price at different levels of service.

ACKNOWLEDGMENTS

The work described in this paper was mainly supported by a grant from the National Natural Science Foundation of China and the National Basic Research Program of China. This study is also partially funded by the Natural Science and Engineering Research Council of Canada.

REFERENCES

1. May, A. D. Integrated Transport Strategies: A New Approach to Urban Transport Policy Formulation in UK. *Transport Reviews*, Vol. 11, No. 3, 1991, pp. 242–247.
2. Zhao, Y. P., and A. Z. Hu. Some Problem in Theoretical Study on Comprehensive Transportation (in Chinese). *Journal of Chongqing Jiaotong Institute*, Vol. 13, No. 2, 1994, pp. 1–6.
3. Liu, X. L., Y. X. Zeng, and Y. M. Su. Some Problems About Development of Integrated Transport in China (in Chinese). *Journal of East China Jiaotong University*, Vol. 13, No. 1, 1996, pp. 90–94.
4. McFadden, D. Conditional Logit Analysis of Qualitative Choice Behavior. In *Frontiers in Econometrics* (P. Zaremka, ed.), Academic Press, New York and London, 1973.

5. McFadden, D. The Measurement of Urban Travel Demand. *Journal of Public Economics*, Vol. 3, 1974, pp. 303–328.
6. Oum, T. H., and Gillen, D. W. *The Structure of Intercity Travel Demands in Canada: Theory, Tests and Empirical Results*. Working Paper No. 79-18. Queen's University, Canada, 1979.
7. Si, B. F., and Z. Y. Gao. Sensitivity Analysis for the Relationship Between Passenger Traffic and Passenger Volume of Railway (in Chinese). *Journal of the China Railway Society*, Vol. 21, No. 4, 1999, pp. 13–16.
8. Si, B. F., and Z. Y. Gao. Optimal Model and Solution Algorithm for Railway Passenger-Ticket Pricing Problems (in Chinese). *Journal of Management Science in China*, Vol. 4, 2001, pp. 45–51.
9. *High Speed Ground Transportation of America*. FRA, U.S. Department of Transportation, 1997.
10. *Final Report on the National Maglev Initiative*. FRA, U.S. Department of Transportation, 1993.
11. *Report to Congress: Costs and Benefits of Magnetic Levitation*. FRA, U.S. Department of Transportation, 2005.
12. Abdulaal, M., and L. J. LeBlanc. Continuous Equilibrium Network Design Models. *Transportation Research Part B: Methodological*, Vol. 13, No. 1, 1979, pp. 19–32.
13. Yang, H., and M. G. H. Bell. Traffic Restraint, Road Pricing and Network Equilibrium. *Transportation Research Part B: Methodological*, Vol. 31, No. 4, 1997, pp. 303–314.
14. Hong, K., and W. Y. Szeto. A Methodology for Sustainable Traveler Information Services. *Transportation Research Part B: Methodological*, Vol. 36, No. 2, 2002, pp. 113–130.
15. Sheffi, Y. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Prentice Hall, Englewood Cliffs, N.J., 1985.
16. Fisk, C. S. Game Theory and Transportation Systems Modeling. *Transportation Research Part B: Methodological*, Vol. 18, No. 4, 1984, pp. 301–313.
17. Suwansirikul, C., T. L. Friesz, and R. L. Tobin. Equilibrium Decomposed Optimization: A Heuristic for the Continuous Equilibrium Network Design Problem. *Transportation Science*, Vol. 21, No. 4, 1987, pp. 254–263.
18. Tobin, R. L., and T. L. Friesz. Sensitivity Analysis for Equilibrium Network Flows. *Transportation Science*, Vol. 22, No. 4, 1988, pp. 242–250.
19. Friesz, T. L., A. Anandalingham, N. J. Mehta, K. Nam, S. J. Shah, and R. L. Tobin. The Multiobjective Equilibrium Network Design Problem Revisited: A Simulated Annealing Approach. *European Journal of Operational Research*, Vol. 65, No. 1, 1993, pp. 44–57.
20. Yang, H., and S. Yagar. Traffic Assignment and Traffic Control in General Freeway-Arterial Corridor Systems. *Transportation Research Part B: Methodological*, Vol. 28, No. 6, 1994, pp. 463–486.
21. Yang, H. Sensitivity Analysis for Queuing Equilibrium Network Flow and Its Application to Traffic Control. *Mathematical and Computer Modeling*, Vol. 22, No. 4, 1995, pp. 247–258.
22. Stephen, D. C., and P. W. David. Sensitivity Analysis of the Probit-Based Stochastic User Equilibrium Assignment Model. *Transportation Research Part B: Methodological*, Vol. 36, 2002, pp. 617–635.
23. Chan, K. S., and W. H. K. Lam. Optimal Speed Detector Density for the Network with Travel Time Information. *Transportation Research Part A: Policy and Practice*, Vol. 36, No. 3, 2002, pp. 203–223.
24. *The Passenger Transportation Market Investigation Between Beijing and Tianjin* (in Chinese). Beijing Railway Bureau, 2000.

The Intercity Passenger Rail Committee sponsored publication of this paper.