# Unsteady State Heat Transfer

## Introduction

Although most mathematical models are derived based on steady-state assumptions, the transient behavior of physical and chemical processes is often of great importance. The engineer must consider such things as reaction mechanisms, transport phenomenon, thermal lag, etc…to operate and optimize their processes. Mathematical models are often derived from first principle balances of mass and energy and can have many dependent variables. Understanding the process practically and theoretically, the engineer is often able to make assumptions to simplify the analysis. One simple experiment to explore these ideas is unsteady-state cooling.

By measuring heat transfer rates vary for metallic, cylindrical rods of different geometry and material, this experiment will show how engineers may make assumptions to simplify their mathematical analysis.

## Objectives:

- to study the rates of heat transfer for different materials and geometries
- to understand the importance and validity of engineering assumptions through the lumped heat capacity method.
- to develop a good understanding of the concepts of forced and free convection

## Theory

A study of the heat losses occurring when an object is suddenly immersed in an infinite, constant temperature heat sink will produce a first order differential equation with temperature as a function of time. Solution of this model can be exact given an initial boundary condition, or can be approximate if the *lumped heat capacity* model is applicable.

This model assumes that the surface temperature of a convectively cooled component is very close to its centre-line temperature (within 5%). It suggests that the resistance due to conduction from the center to the surface is negligible compared to the resistance of convection to the surroundings. Hence, the combined resistance is "lumped" together without great error. The exact model assumes that the temperature profile across a cooled solid is not negligible from center to surface.

To determine the applicability of a model, the *Biot Number* is used. It is a unitless ratio of an object's internal conduction resistance to it's external convective resistance. If the value of Bi is less than 0.1, then the lumped heat capacity model is applicable. Otherwise, the exact solution is necessary.

Lumped Heat Capacity Model:

$$\begin{aligned} q &= \frac{T - T_e}{T_0 - T_e} \\ \ln(q) &= \frac{h_c A t}{r V C_p} \end{aligned} \tag{4}$$

Exact Method:

$$-\ln q = l^2 Fo + \ln A \tag{5}$$

where,

$$Fo = \frac{kt}{r\,C_p R^2} \qquad (6)$$

By plotting this equation (-ln2 vs. Fo), the slope gives $8^2$. This value can be used to determine the Biot number by interpolating the values in Table 3.5 [4], which can then be used to calculated the heat transfer coefficient. These topics are covered in depth in ChE 3304 [ref 1].

## Apparatus:

This experiment uses a two large, insulated, water-filled basins – one heated, the other at room temperature. A small circulating pump is forces the water through a screen to streamline the flow profile evenly across the objects immersed for cooling. The rods are equipped with T-type thermocouples, mounted on the centerline of each, and a PC is provided to record the temperature as a function of time. Specifics of the rods are as follows:

| | Mild Steel rods | Copper rods | Stainless Steel rods | Teflon rod | Teflon sphere |
|---|---|---|---|---|---|
| **Density ($kg/m^3$)** | 7840 | 8933 | 7900 | 2200 | 2200 |
| **Heat Capacity (J/kg K)** | 460 | 385 | 477 | 1050 | 1050 |
| **Thermal Conductivity (W/mK)** | 50 | 401 | 15 | 0.45 | 0.45 |

**The radii of the rods are either 0.83 or 1.65 cm and 30.5 cm in length.**

References:

[1] Mills, A. F. Heat Transfer - Second Edition. Prentice Hall. Upper Saddle River, NJ. 1999.

[2] Incropera, F.P. and D.P. DeWitt, Fundamental of Heat and Mass Transfer, 2nd ed. John Wiley and Sons Inc. New York (1985).

[3] A.F. Mills, Heat Transfer, Richard D. Irwin Inc. Boston, (1992).

[4] Brodkey, R.S., Hershey, H.C., Transport Phenomena. A unified approach, McGraw Hill Book Co., New York (1988).

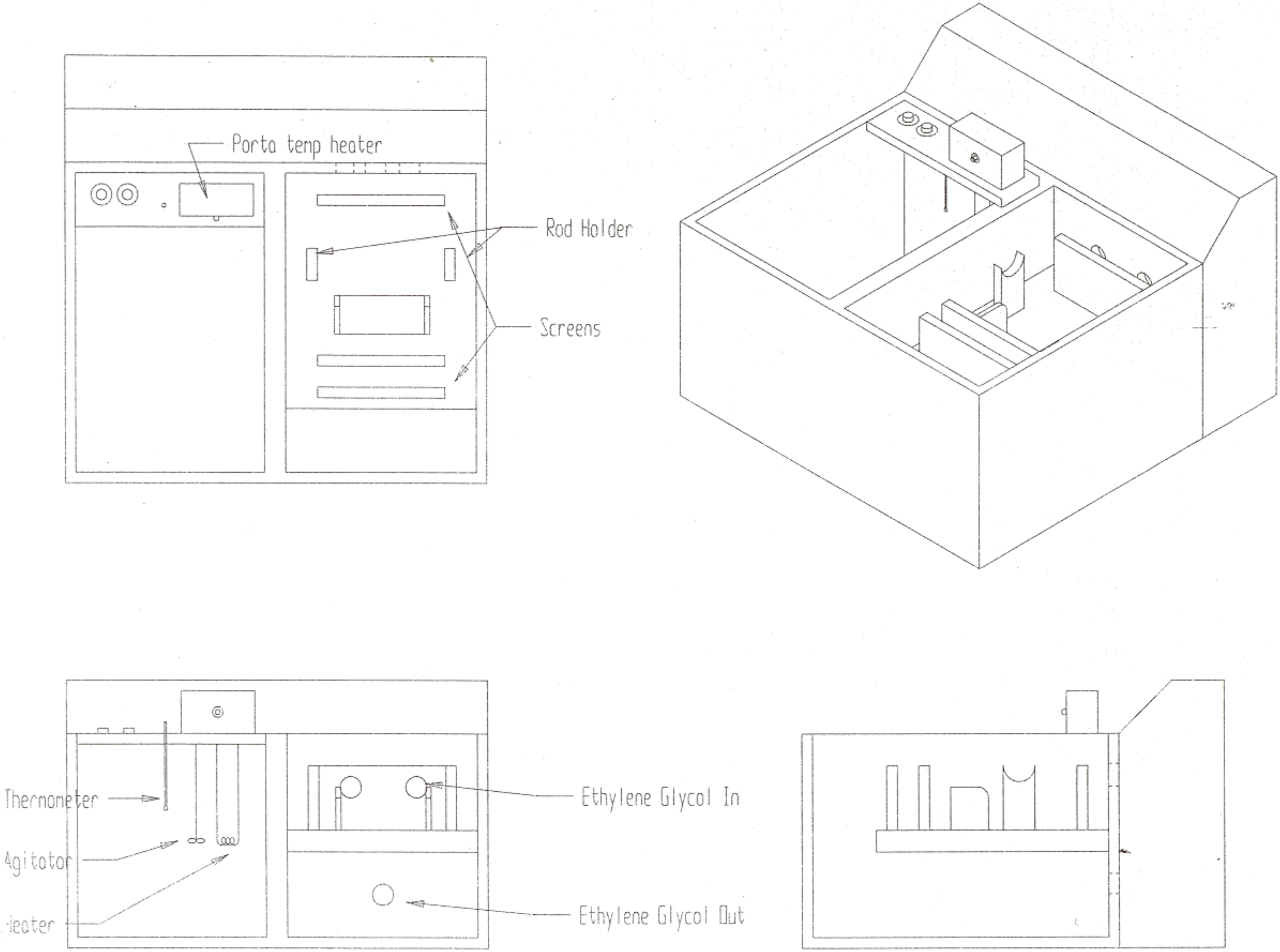

Figure 1: Schematic for USHT Apparatus